STUDY OF LINEAR COUPLING COMPENSATION FOR THE STORAGE RING OF SPRING-8

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Abstract

Linear coupling compensation for the storage ring of SPring-8 under the practical error condition is described. It is found that the coupling effect is so strong that we can not achieve the ring design performance without some coupling compensation scheme, and that the method with only two skew quadrupoles is useful to the coupling compensation to some extent.

Introduction

The storage ring of SPring-8 is a Chasman Green type low emittance ring. In this kind of storage rings, strong quadrupoles and sextupoles are usually used in order to strongly focus the beams and to compensate large chromaticities. In the case that these magnets are tilted and misaligned, they make the circulating beams feel the strong skew quadrupole error fields. Therefore, the strong linear coupling of lateral beam oscillations is suspected in the storage ring of SPring-8.

On the other hand, it is required for the various experiments by the photon beam users that the coupling ratio of vertical and horizontal emittance is adjustable from 1 % to about 100 % (full coupling condition). In addition, it is crucial to suppress the increase of vertical emittance lower than 1 mm for the stable beam injection with high efficiency.

For these requirements, the linear coupling should be compensated enough to obtain 1 % coupling ratio. In respect to the increase of the coupling ratio, it can be easily controlled with the adjustment of the workpoint.

Linear coupling theory

A coupling coefficient κ_q, which is derived from Hamiltonian function to include a skew quadrupole potential through a canonical transformation and Fourier expansion, is used to estimate the strength of the linear coupling. The coupling coefficient κ_q is expressed as[1,2]

$$\kappa_q e^{i\phi_q} = \frac{1}{2\pi}\int_{\theta}^{\theta+2\pi} d\theta k_s \sqrt{\beta_x \beta_y}\, e^{i[\Psi_x - \Psi_y - (Q_x - Q_y + q)\theta]}, \qquad (1)$$

where Ψ_i and β_i (i = x : horizontal plane, y : vertical plane) are Floquet phase and the betatron function, Q_i and ϕ_q are the betatron tune and the phase of q harmonics corresponding to the strength of sine and cosine terms of κ_q, k_s and L are the strength of skew quadrupole components and the ring circumference, respectively, and $\theta = 2\pi s/L$ (s : azimuthal length, L : ring circumference) denotes the time variable. The parameter q represents the differential resonance condition and the resonance to be compensated often satisfies the condition that δ ($\delta = Q_x - Q_y - q$) is minimum. The parameter δ denotes the distance from the resonance.

The horizontal and vertical emittance are related to κ_q through perturbation treatment of the Hamiltonian function and the resonance approximation as follows.

$$r = \kappa_q^2/(\kappa_q^2 + \delta^2), \qquad (2)$$

where r is the coupling ratio of vertical and horizontal emittance.

Aspect of linear coupling in the storage ring of SPring-8

The expectation value of κ_q is calculated by well known statistic treatment[3] of Eq. (1). In this calculation, a tilt error of a quadrupole and vertical offset errors of a sextupole, which are the first order of the error perturbation, are assumed as a source of a skew quadrupole component. The offset errors mean a misalignment of a sextupole and closed orbit distortion (COD) at a sextupole. For the normal operation mode (hybrid mode)[4] in which the nearest resonance to the workpoint is $Q_x - Q_y = 32$, the expectation value of κ_{32} is calculated as,

$$[\kappa_{32}]_{tilt} = 34.58\,[\theta], \qquad (3)$$

$$[\kappa_{32}]_{offset} = 208.88\,[\Delta z], \qquad (4)$$

$$[\kappa_{32}]_{total} = \sqrt{[\kappa]_{tilt}^2 + [\kappa]_{ofset}^2}, \qquad (5)$$

where θ and Δz are the tilt error angle and the ofset error, respectively, and [p] denotes the root mean square value of a parameter p. From above formulae, it is found that the contribution from sextupoles is dominant in the storage ring of SPring-8. This is general for the low emittance ring where lots of strong sextupoles are installed. It is suspected that the root mean square value of the alignment error, the residual COD, and the tilt error is 0.2 mm, 0.1 ~ 0.15 mm, and 0.5 mrad, respectively. The expectation value of κ_{32} for the storage ring of SPring-8 is estimated to be 0.05 ~ 0.055.

The coefficient κ_q can be also calculated with the vertical amplitude beating of the tracking particle which is started with zero vertical emittance. The values of κ_{32} for 10 rings are distributed from 0.006 ~ 0.07 and the average value is about 0.03 which is slightly lower than the expectation value. This may be because the errors of tracking simulation are cut off at the double root mean square value. In this paper, it is presumed that the expectation value of κ_{32} exists in the range from 0.03 ~ 0.055.

With κ_q and δ, the coupling ratio is estimated by Eq. (2). The coupling ratio is shown as a function of κ_q and as a parameter of δ in Fig. 1. The parameter δ is 0.06 at the normal hybrid mode. The maximum value of δ is limited to

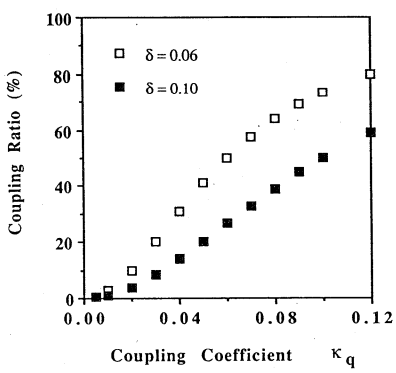

Fig. 1 Coupling ratio v.s. coupling coefficient κ_q

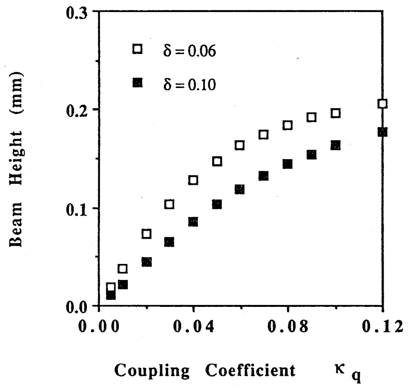

Fig. 2 Beam height v.s. coupling coefficient κ_q
Horizontal emittance is 5.3 nmrad and beam width is 0.33 mm and β_y is 10 m.

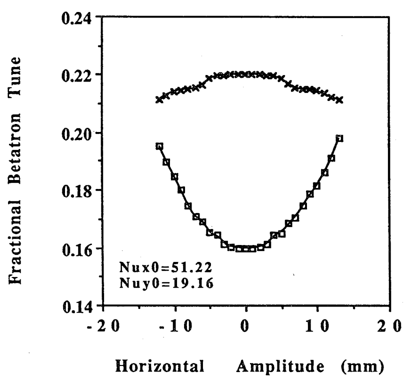

Fig. 3 Amplitude dependent tune shift diagram
Cross represents vertical tune and white square represents horizontal tune. Vertical amplitude ratio is 1 %.

about 0.1, because the operation range of the workpoint is limited by the first and third order resonances from the viewpoint of dynamic stability of the ring. With the parameter condition mentioned above, the expectation value of coupling ratio becomes 20 ~ 50 % at the normal workpoint, $\delta = 0.06$ and 10 ~ 20 % at $\delta = 0.1$. That is , it is not foreseen that the linear coupling can be controlled with only workpoint tuning. The beam height at the high β straight section is shown as a function of κ_q in Fig. 2 and from that Figure, it is estimated that the expectation value of beam height becomes 0.1 ~ 0.15 mm at $\delta = 0.06$ and 0.06 ~ 0.11 at $\delta = 0.1$.

The situation of the coupling at the large amplitude can be investigated with the amplitude dependent tune shift diagram shown in Fig. 3. It is observed that the increase of the horizontal amplitude makes δ small. This fact means that the coupling ratio of vertical and horizontal emittance becomes larger at the beam injection than at the steady state after a radiation damping. An example of a phase space map at the injection are shown in Fig. 4. In this simulation, the beam are injected into the ring with small vertical emittance which is negligible compared to the horizontal one (horizontal initial amplitude = 8 mm), but the vertical oscillation is excited by the skew quadrupole fields in the ring and vertical emittance becomes quite large after dozens of revolutions. We can find that particle motion in Fig. 4 is almost full coupling through the divergence of vertical phase space. That fact is also confirmed by the coincidence of both tunes measured by the Fourier analysis. The coefficient of κ_{32} of this ring is about 0.025, which is not so large.

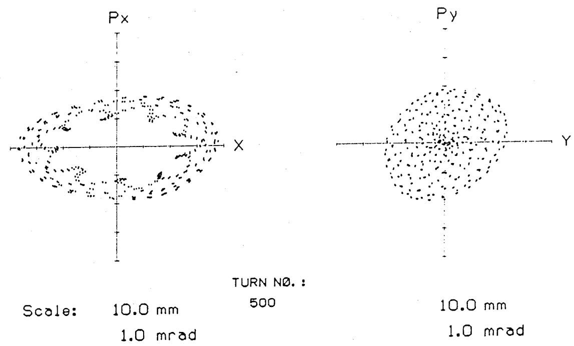

Fig. 4 Phase space plot at injection simulation

Linear Coupling compensation scheme

In order to compensate κ_q for the arbitrary operation mode, all terms (cosine and sine terms) in right hand of Eq. (1) should be vanished with skew quadrupole magnets. The linear coupling phase part in right hand of Eq. (1) is represented here with Φ_q and it is assumed that a set of only two skew quadrupoles, S1 and S2, are installed at Φ_{q1} and Φ_{q2}, respectively. The compensation can be perfectly achieved with this set, if Φ_{q1} and Φ_{q2} satisfy the following relation[5],

$$\Phi_{q1} + \Phi_{q2} = (2\pi / 4) \times n, \qquad (6)$$

where the subscript 1 and 2 represent the parameter of S1 and S2, n is arbitrary odd integer. But actually, quite large numbers of skew quadrupoles should be introduced to obtain two degrees of freedom for the compensation totally, because it is difficult to find the set of two skew quadrupoles satisfying the above relation for all operation modes.

On the other hand, the storage ring of SPring-8 is characterized by its large cell periodicity, maximum 48 periodicity. With this large periodicity and the easy constraints to operation flexibility, we can find the set of only two skew quadrupoles satisfying Eq. (6). The phase difference between two skew quadrupoles separated at n_{12} cells are represented with the superperiodicity N as,

$$\Phi_{q1} = \Psi_{x1} + \Psi_{y1} - (Q_x - Q_y - q) \times \theta_1, \qquad (7)$$
$$\Phi_{q2} = \Psi_{x2} + \Psi_{y2} - (Q_x - Q_y - q) \times \theta_2, \qquad (8)$$
$$\Delta\theta = \theta_2 - \theta_1 = 2\pi \times (n_{12} / N), \qquad (9)$$
$$\Delta\Phi = \Phi_{q1} - \Phi_{q2} = q \times (\theta_2 - \theta_1)$$
$$= 2\pi \times q \times (n_{12} / N). \qquad (10)$$

If the workpoint can be determined as the combination of odd and even number in integer part of betatron tune, we can always make q some odd number. Furthermore, if N of all operation modes can be limited a multiple of 4, we make n_{12} / N equal to 1/4 for all operation modes by the arrangement where two skew quadrupoles are separated with the quadrant of the ring. Consequently, two skew quadrupoles correctly satisfy Eq. (6). In the operation of the storage ring of SPring-8, the minimum superperiodicity is 4 at the operation mode with 4 long straight sections. The constrain to superperiodicity is not serious. As concerns the constrain to the workpoint, it is not either serious. The workpoint of normal hybrid mode is now composed of two odd numbers in integer part, but it is easily adjusted with vertical detuning without reduction of the dynamic stability.

With the above criteria, the relative positions of two skew quadrupoles can be determined. Next, the specified position in a cell for the skew quadrupoles should be selected. On the selection, following two things should be considered. One is to select the position in the dispersion free section. If the skew quadrupoles are located in the dispersive section, the vertical spurious dispersion is induced by the linear coupling and vertical emittance becomes large. The other is to select the position with high β_x and β_y. From Eq. (1), it is found that κ_q is proportional to the square root of β_x and β_y. The strength of the skew quadrupoles can be reduced with high β_x and β_y. Considering these requirements, the position at the harmonic sextupoles in dispersion free section and at the edge of dispersion free straight section are selected as candidates.

Computer simulation

The normal hybrid mode is used for the simulation of linear coupling compensation. As the workpoint of this mode does not satisfy the even-odd combination, we can not use the scheme described here directly. In order to investigate the effect of two skew quadrupoles which satisfy Eq. (6) exactly, we rearrange two skew quadrupoles for the normal hybrid mode, regardless periodicity. This is enough to know whether our scheme is useful or not.

The subroutine for the compensation simulation is added to the tracking program RACETRACK[6] RIKEN version. The simulation is performed according to the following procedure. At first the COD is corrected at the level of the residual COD lower than 0.15 mm for the ring including normal errors (the misalignment error 0.2 mm, the tilt error 0.5 mrad, the field error 5×10^{-4}, the gradient error 5×10^{-4}). After the COD correction, the strength of the skew quadrupoles are fitted to minimize the maximum vertical amplitude of the tracking particle in 50 revolutions. As the COD and Twiss parameter are distorted during the fitting, the COD and the Twiss parameter calculation, the COD correction can be used in fitting loop to cancel the fitting error. Mapping of the coupling ratio as a function of skew quadrupole strength can be used to determine the initial strength.

The result of the simulation is shown in Table 1 and An example of improvement of the coupling is shown in Fig. 5. The simulation is performed for 4 rings of which κ_{32} is relatively large among 10 rings mentioned before and remarkable improvement can be found in the coupling ratio. The coupling ratio is reduced to one third ~ one fifteenth of the initial value; 4 ~ 10 % by the compensation. The maximum strength of skew quadrupoles is about 0.023 m^{-1}.

Although it is difficult to find the reason why the coupling can not be compensated completely, we guess following two things. One is local phase distortion by the ring errors between two skew quadrupoles which breaks the indispensable phase relation. The other is the problem of fitting performance which means the program can not search an optimum strength from the arbitrary initial condition.

Table 1

simulation result of the coupling compensation

No	Before compensation		After compensation	
	κ	Coupling ratio (%)	κ	Coupling ratio (%)
1	0.070	53	0.012	3.9
2	0.039	30	0.015	6.5
3	0.039	30	0.021	11
4	0.047	39	0.020	10

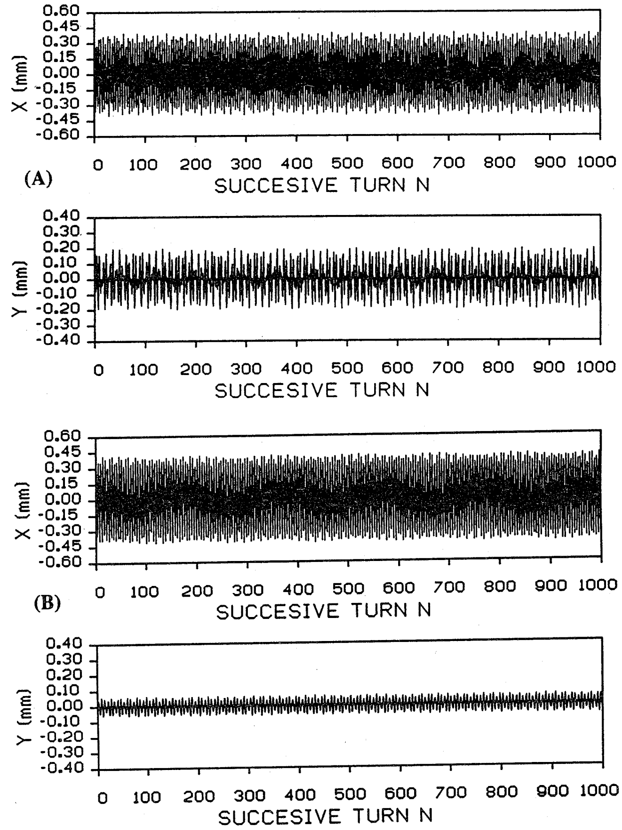

Fig. 5 Horizontal and vertical emittance beat

This is the result of linear coupling compensation for No 1 in Table 1. X and Y represent horizontal and vertical oscillation amplitude. (A) and (B) denote before and after compensation.

Summary

The situation of the linear coupling was investigated. It is found that the expectation value of linear coupling coefficient is 0.03 ~ 0.055 and that the coupling ratio is 20 ~ 50 % at the normal workpoint. These values are too large to control the coupling ratio with workpoint adjustment. In order to satisfy requirement that the coupling ratio is adjustable in wide range (1 % ~ about 100 %), some compensation scheme should be used.

The compensation scheme with only two skew quadrupoles was proposed and the computer simulation was performed to study its effect. It is found that the compensation scheme is effective to some extent, but can not satisfy our requirement, 1% coupling ratio. Detail investigation of this imperfect compensation and another compensation scheme based on its result should be pursued.

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