TRANSVERSE BEAM BUNCHING METHOD
USING A DEFLECTING MODE

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Abstract

A transverse bunching method is presented for an electron beam using a deflecting mode in a non-axisymmetric RF cavity. The method's principle is based on an interaction between synchrotron and betatron oscillations. A damping of the betatron oscillation leads to a reduction of beam size at the expense of growth in energy spread. The damping time can be controlled by varying the amplitude of the deflecting mode or by detuning of the deflecting mode frequency from an acceleration frequency.

Introduction

In an electron synchrotron or storage ring, transverse beam bunching usually depends only on radiation damping or adiabatic damping. Here we present a transverse beam bunching method by electromagnetic fields of a deflecting mode.

The electric and magnetic fields affect betatron and synchrotron oscillations. The electric field affects the synchrotron oscillation through an energy change, and it affects the betatron oscillation through a change of a closed orbit. The magnetic field affects the betatron oscillation through momentum kick, which in turn affects the synchrotron oscillation through a change of revolution time. In the method, a betatron oscillation amplitude is damped through a coupling of synchrotron and betatron oscillations.

This method can be applied to injection of a low energy beam so that the beam is compressed within a good field region to reduce beam loss. Another application is found in reduction of betatron emittance in a high energy electron ring.

Synchro-Betatron Oscillation

The synchrotron and betatron oscillations have interactions due to the following factors,

(1) energy dispersion of a closed orbit at RF cavities,
(2) chromaticity,
(3) magnetic fields in RF cavities,
(4) transverse dependence of electric fields in RF cavities,
(5) change of revolution time due to the betatron oscillation, and
(6) wake fields (transient electromagnetic field caused by the beam-cavity interaction).

The synchrotron oscillation affects the betatron oscillation through factors (1), (2) and (3), while the betatron oscillation affects the synchrotron oscillation through factors (4) and (5). Through factor (6), the synchrotron and betatron oscillations affect themselves and interact with each other. This is the main cause of beam instability. The chromaticity in factor (2) is usually corrected to zero to suppress the head-tail instability of the betatron oscillation, so this factor can be neglected here. Furthermore the wake fields in factor (6) are assumed to be neglected.

Basic Equations

In order to analyse synchro-betatron motion, basic equations are presented for numerical simulations. The betatron motion is assumed to be linear, except for RF cavities. An accelerating cavity and a deflecting mode cavity are installed in a straight section of the ring. For convenience, a normalized phase space (x, y) is used for the betatron motion

$$y = \alpha x + \beta x', \tag{1}$$

where α and β are Twiss parameters.

The accelerating and deflecting mode cavities are designated as ACC and TBB, respectively, and the actions of the RF fields are treated like a delta-function. We use the betatron phase space (x, y) and the synchrotron phase space (θ, δ) where θ is an RF phase and δ is a energy deviation defined as

$$\delta = \frac{E - E_0}{E_0}, \tag{2}$$

with E as electron energy and E_0 as synchronous electron energy.

The basic equations are described as follows[1,2] ;

(1) At ACC and TBB

$$\Delta x = -\eta \Delta \delta_p, \tag{3}$$

$$\Delta y = \frac{1}{1+\delta_p^+}[(\delta_p^+ - \delta_p^-)\times \times\{\delta_p^- \alpha\eta - (1+\delta_p^+ + \delta_p^-)\zeta + (\alpha x_- - y_-)\} + \beta\phi_H], \tag{4}$$

$$\Delta\theta = 0, \tag{5}$$

$$\Delta\delta = \frac{eV(x+\eta\delta, \theta)}{E_0}, \tag{6}$$

where

$$\delta_p = \frac{p - p_0}{p_0}, \tag{7}$$

with p as electron momentum and p_0 as synchronous electron momentum.

(2) From ACC to TBB

$$\begin{pmatrix} x_T \\ y_T \end{pmatrix}_- = \sqrt{\frac{\beta_T}{\beta_A}} \begin{pmatrix} \cos\mu_{AT} & \sin\mu_{AT} \\ -\sin\mu_{AT} & \cos\mu_{AT} \end{pmatrix} \begin{pmatrix} x_A \\ y_A \end{pmatrix}_+, \tag{8}$$

$$\Delta\theta = 0, \tag{9}$$

$$\Delta\delta = 0. \tag{10}$$

(3) From TBB to ACC

$$\begin{pmatrix} x_A \\ y_A \end{pmatrix}_- = \sqrt{\frac{\beta_A}{\beta_T}} \begin{pmatrix} \cos\mu_{TA} & \sin\mu_{TA} \\ -\sin\mu_{TA} & \cos\mu_{TA} \end{pmatrix} \begin{pmatrix} x_T \\ y_T \end{pmatrix}_+, \tag{11}$$

$$\Delta\theta = 2\pi h \alpha_m \delta + \frac{k_0}{\beta_T}(a x_T + b y_T)_+, \tag{12}$$

$$\Delta\delta = 0. \tag{13}$$

where

$$k_0 = 2\pi \frac{f_0}{c}, \tag{14}$$

f_0: acceleration frequency,
c : velocity of light.

In these equations, the following parameters are used.

β_A: betatron function at ACC,
β_T: betatron function at TBB,
μ_{AT}: betatron phase advance from ACC to TBB,
μ_{TA}: betatron phase advance from TBB to ACC,
h : harmonic number,

α_m : momentum compaction factor,

$$a = \eta_T \sin\mu - \zeta_T (1-\cos\mu), \quad (15)$$

$$b = \eta_T (1-\cos\mu) + \zeta_T \sin\mu, \quad (16)$$

$\mu = \mu_{AT} + \mu_{TA}$: betatron phase advance per turn, (17)

$$\zeta_T = \alpha_T \eta_T + \beta_T \eta_T' . \quad (18)$$

The symbol Δ denotes a change of the following value. The subscripts − and + in Eqs.(4),(8),(11) and (12) indicate values just before and after each cavity. Equation (12) was derived by Piwinski and Wrulich[1,3]. In Eq.(12), the first term including δ is a phase shift by the change of the circumference due to energy dispersion of the closed orbit, while the second term, including x and y, is due to the betatron oscillation.

Deflecting Mode

The electromagnetic effects of the deflecting mode are described by using an electric voltage V and a "magnetic voltage" V_H defined as

$$V = \int E_s \, ds, \quad (19)$$

$$V_H = Z_0 \int H_z ds, \quad (20)$$

E_s: longitudinal electric field,
H_z: vertical magnetic field,
Z_0: characteristic impedance in a vacuum.

The change of the energy deviation $\Delta\delta$ and the momentum kick ϕ_H by the RF magnetic field are written as

$$\Delta\delta = -\frac{eV\cos\theta}{E_0}, \quad (21)$$

$$\phi_H = \frac{eV_H \sin\theta}{E_0}. \quad (22)$$

The Maxwell equation relates the electric and magnetic volatages V and V_H as follows,

$$\frac{\partial V}{\partial X} = kV_H, \quad (23)$$

where

$$k = 2\pi\frac{f}{c}, \quad (24)$$

and f is an operating frequency of the deflecting mode. Therefore the two voltages are written in the following form,

$$V_H(X) = V_{HO}(1+gX^2), \quad (25)$$

$$V(X) = kV_{HO}(X + \frac{g}{3}X^3), \quad (26)$$

where X is the horizontal displacement from a central axis of the deflecting mode cavity. "Magnetic impedance" Z_H is conveniently defined as

$$Z_H = \frac{V_{HO}^2}{P_W}, \quad (27)$$

where P_W is power loss of the deflecting mode on the wall.

The deflecting mode cavity having a non-axisymmetric structure was numerically analysed by a three-dimensional computer code for electromagnetic field analysis based on the boundary element method. Figure 1 shows the structure of the deflecting mode cavity. In Fig. 2, the RF magnetic fields of the TM_{110} mode are indicated by arrows for one-fourth of the cavity. The resonant frequency is 150MHz, and we obtained the magnetic impedance Z_H of 6.9MΩ with copper wall, and the coefficient g of 38 m^{-2} in Eq.(25).

Numerical Simulation Results

The synchro-betatron motion was simulated by using the data of the deflecting mode described above. Table 1 lists basic parameters used here.

The basic equations described above satisfy the symplectic conditions for a conservative system, so that the number of turns can be set on the order of one million in the double-precision calculation. However, higher order terms of x and y are neglected in the above basic equations.

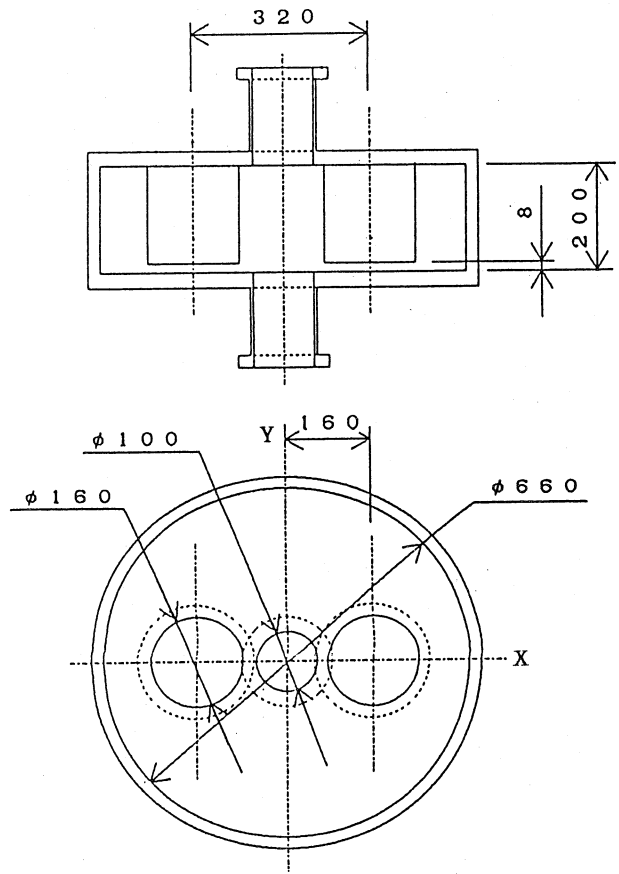

Fig.1 Structure of the deflecting mode cavity.

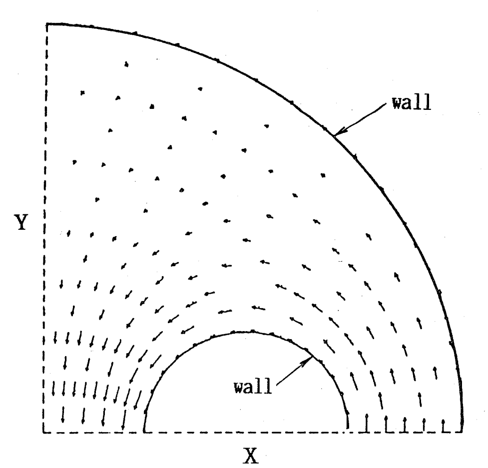

Fig.2 RF magnetic field of TM_{110} mode in one-fourth of the deflecting mode cavity. The numerical result was obtained by the computer code based on the boundary element method.

Therefore, the number of turns was restricted to two millions.

Figure 3 shows the RF phase relation of accelerating and deflecting modes for $f = f_0$ where the synchronous phase is 180°.

Figure 4 shows the synchrotron and betatron motions and beam size evolution with initial RF phase between 140° and 220°, initial energy deviation of zero, and initial betatron amplitude of 3cm.

When the energy gain by the electric voltage is positive for a positive horizontal displacement around the synchronous phase 180°, the betatron oscillation decays, while the synchroton oscillation gradually grows. The damping time of the betatron oscillation can be controlled by varying the amplitude of the deflecting mode or by detuning of the deflecting mode frequency from the acceleration frequency.

Qualitatively we can say that the electric field of the deflecting mode amplifies the synchrotron oscillation and damps the betatron oscillation, and that the magnetic field of the deflecting mode amplifies the betatron oscillation which attracts the particle toward the synchronous phase to damp the synchrotron oscillation. The effect of electric field on the betatron motion is stronger than that of the magnetic field. That is why the betatron oscillation decays and the synchrotron oscillation grows.

Table 1. Basic Parameters

Energy		E_0	20 MeV
Horizontal Betatron Tune		ν	1.58
Momentum Compaction Factor		α_m	0.138
Harmonic Number		h	10
Twiss Parameters	(ACC)	β	1.3 m
		α	−0.43
	(TBB)	β	3.1 m
		α	−1.3
Energy Dispersion	(ACC)	η	1.06 m
Function		η'	1.01
	(TBB)	η	2.07 m
		η'	1.01
Accelerating Voltage	(ACC)		5 kV
Voltage Gradient	(TBB)		10 kV/m
Acceleration Frequency		f_0	150 MHz

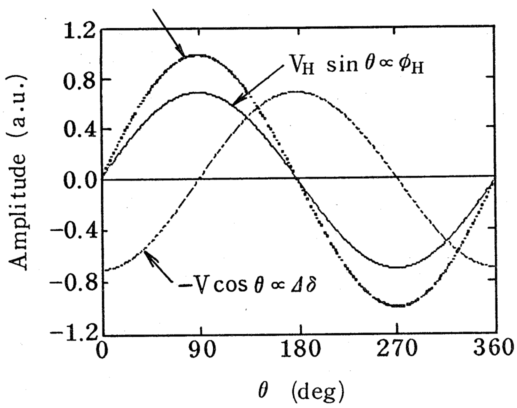

Fig.3 RF phase relation of accelerating and deflecting modes for $f=f_0$. The voltage $-V\cos\theta$ of the deflecting mode is shown for a positive horizontal displacement.

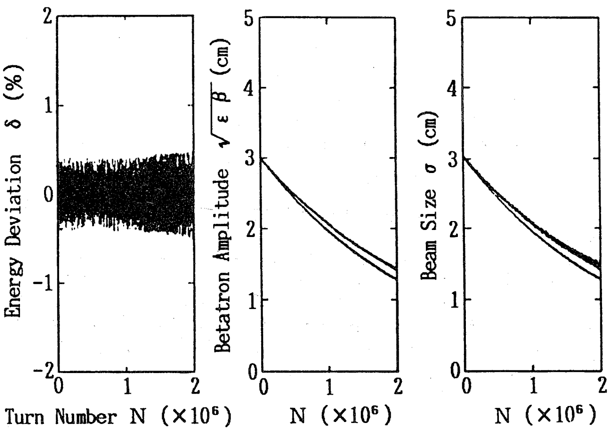

Fig.4 Synchrotron and betatron oscillation amplitude and beam size.

The betatron oscillation grows when the RF phase of the deflecting mode is shifted by 180° from the one shown in Fig. 3. When the initial RF phase is far from the synchronous phase, the particle runs away from the RF-bucket generated by the accelerating voltage.

The RF power of the deflecting mode is calculated by Eq. (27) as 1.5W for the voltage gradient of 10kV/m, while the RF power of the accelerating mode is 25W for the accelerating voltage of 5kV with a shunt impedance of 1MΩ.

Conclusions

We presented a transverse beam bunching method where betatron oscillation can be damped by a deflecting mode at the expense of growth of synchrotron oscillation when the energy gain by the electric voltage of the deflecting mode is positive for a positive horizontal displacement around the synchronous phase.

References

1) A. Piwinski, "Beam Instabilities and Computer Simulations", DESY M-83-21 (1983).
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