FORMULATION OF THE RFQ RADIAL MATCHING SECTION

N. Tokuda and S. Yamada

Institute for Nuclear Study, University of Tokyo
Tanashi, Tokyo 188, Japan

Abstract

The electric field potential function in the RFQ radial matching section has been formulated, adopting the lowest order potential function expanded in a Fourier-Bessel series. The resultant focusing strength increases sinusoidally with the distance along the beam axis. The overlap between the time-dependent RFQ acceptance and the time-independent phase space of the injected beam has been calculated. It has turned out that overlap higher than 90% is available, and that a long radial matcher is preferable to shape the input beam.

1. Formulation of the Potential

The potential of electric field in a four-conductor quadrupole cavity is generally expanded in a Fourier-Bessel series:

$$U(r,z,\psi,t) = \frac{V}{2}\,[F_0(r,\psi) + \sum_{n=1}^{\infty} F_n(r,\psi)\sin nkz] \times \cos(\omega t + \phi) \ , \qquad (1)$$

where

$$F_0(r,\psi) = \sum_{m=0}^{\infty} A_{om}\, r^{2(2m+1)} \cos 2(2m+1)\psi \ , \qquad (2)$$

$$F_n(r,\psi) = \sum_{m=0}^{\infty} A_{nm}\, I_{2m}(nkr) \cos 2m\psi \ , \qquad (3)$$

$$k = \pi/L \qquad (4)$$

The length, L, is a half of the periodic length, which will be determined later. Imposing boundary conditions that the potential is zero on the beam axis (r=0) and the cavity end wall (z=0), and taking quadrupole (m=1) and the lowest order (n=1) terms, we have

$$U(r,z,\psi) = \frac{V}{2}\frac{I_2(kr)}{I_2(ka)} \sin kz \cos 2\psi \ , \qquad (5)$$

$$k = \pi/2\ell \ , \qquad (6)$$

where a is the bore radius at the output of the radial matcher, and ℓ is the length of this section, which is set at a half of L to obtain smooth connection to the RFQ structure. The focusing strength is derived from the coefficient of the linear term of the radial component of the electric field,

$$B(z) = \frac{1}{8}\frac{k^2 a^2}{I_2(ka)} B_0 \sin kz \qquad (7)$$

$$\simeq B_0 \sin kz \qquad (8)$$

The vane of the radial matcher is shaped to an equipotential surface given by Eq. (5). Figure 1 shows the vane cross section and electric field lines in the x-z plane at the INS RFQ 'LITL'[1]. The vane is chopped at z = 0.4 cm, so that the electric field in the beam passing region (r ≲ 0.25 cm) may not change much.

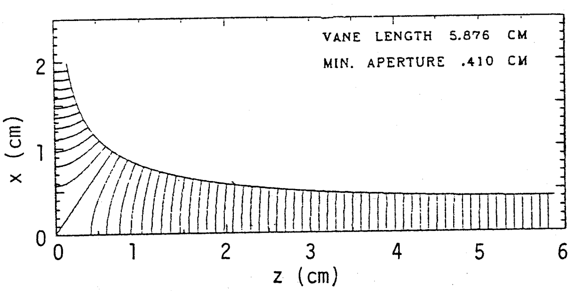

Fig. 1. Vane cross section and electric field lines in the radial matcher of LITL.

2. Phase Space Matching through the Radial Matcher

The ellipse parameters of the RFQ acceptance are obtained with a transfer matrix through one focusing unit (two cells) of constant focusing strength of B_0. The acceptance rotates with rf phase as shown in Fig. 2. The ellipse parameters of the injected beam to be matched with the RFQ acceptance are derived using a transfer matrix through the radial matcher, which is calculated with the sinusoidally increasing focusing strength. Figure 3 shows such input beam phase space ellipses matching through a 12 cell long radial matcher of LITL. As is apparent, the ellipse parameters are almost independent of rf phase, and an ellipse can be drawn over the phase space ones so as to attain overlap higher than 90 %.

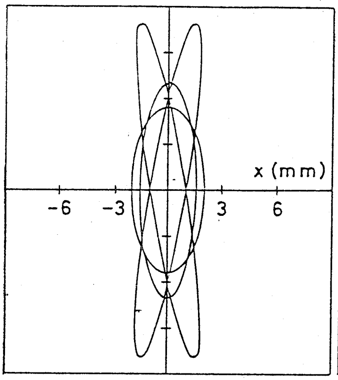

Fig. 2. Phase space ellipses of the RFQ acceptance in the case of LITL. B_0 = 5 and ε_n = 0.6 π mm·mrad (v/c = 0.00326).

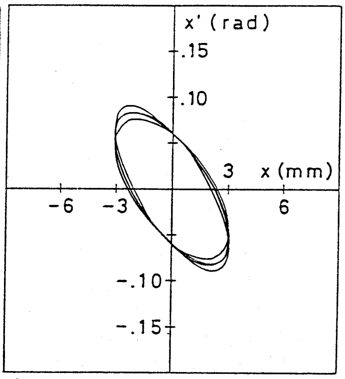

Fig. 3. Phase space ellipses at the input of the radial matcher. They are matched with the RFQ acceptance ellipses in Fig. 3 through a radial matcher of 12 cells.

Although a shorter radial matcher might do from the point of view of overlap, a long radial matcher is preferable, as the ellipse parameters of the matching input beam phase space varies with the length of the radial matcher. As is shown in Fig. 4, with increase of the length, the size of the input beam gets large and its gradient small, i.e. the beam shaping gets easier. As the cell length is short in the radial matcher (0.49 cm at LITL), a large number of cells does not cost length.

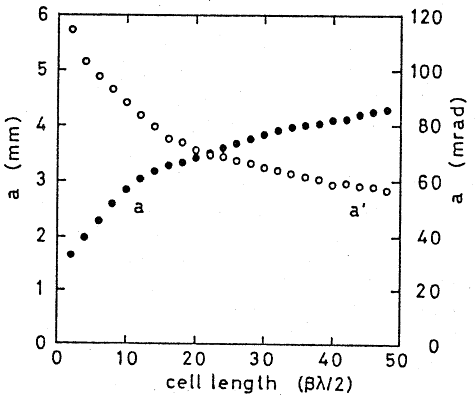

Fig. 4. Dependence of the size of the input beam ($a = \sqrt{\beta\varepsilon/\pi}$) and its gradient ($a' = \sqrt{\gamma\varepsilon/\pi}$) on the length of the radial matcher. Focusing strength and input beam emittance are same as in Fig. 2.

Acknowledgments

The main part of this work was done at Lawrence Berkeley Laboratory, University of California, USA. The authors express their sincere thanks to Drs. J. Staples and R. Gough for their encouragement and discussions through the work.

References

1. N. Ueda et al., An RFQ Linac for Heavy Ion Acceleration, 1981 Linear Accelerator Conference, Santa Fe, NM, USA, 1981. Papers presented at this symposium.